MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2019/2020

PCM0235 - CALCULUS

(Foundation in Information Technology/ Foundation in Life Sciences)

12 OCTOBER 2019
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This Question paper consists of 2 pages excluding the cover page and appendix.
2. Answer **all FIVE** questions. Each question carries equal marks and the distribution of the marks is given.
3. Write all your answers in the Answer Booklet provided.

Question 1 **[10 marks]**

a. Find the values of a and b so that $f(x)$ is differentiable everywhere.

$$f(x)=\begin{cases}1-ax+bx^2, & x\le -1\\ x^2+x, & -1\le x<2\\ ax^2+bx+4, & x\ge 2\end{cases}$$ (4 marks)

b. Find the following limits.

i. $\lim\limits_{x\to\infty}\dfrac{x^2+x+1}{(3x+2)^2}$ (3 marks)

ii. $\lim\limits_{t\to 0}\dfrac{\sqrt{t^2-t+4}-2}{t^2+3t}$ (3 marks)

Question 2 **[10 marks]**

a. Differentiate the following functions.

i. $y=\sin\left(\ln\left(5x^2-2x\right)\right)$ (2 marks)

ii. $(x-y)^2=x+y-1$ (3 marks)

b. Use integration by parts to evaluate $\int x^2e^{3x}dx$ (5 marks)

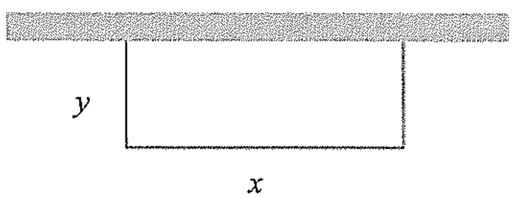
Question 3 **[10 marks]**

a. Given $f(x)=(x+3)(x-2)^3$. Find

i. the interval(s) of increase and decrease of f and the value(s) of x for which f has local maximum and minimum. (3 marks)

ii. the interval of concavity and inflection point(s). (3 marks)

b. A rectangular garden is to be constructed using a rock wall as one side of the garden and wire fencing for the other three sides. Given 100 ft of wire fencing, determine the dimensions that would create a garden of maximum area. What is the maximum area?



(4 marks)

Continued…

Question 4 [10 marks]

a. Find the area of the region enclosed between the curves defined by the equations $y = x^2 - 2x + 2$ and $y = -x^2 + 6$. (5 marks)

b. Find the volume of the solid generated by rotating the region bounded by $y = \sqrt[3]{x}$ and $y = \frac{x}{4}$ that lies in first quadrant about the y-axis. (5 marks)

Question 5 [10 marks]

a. Find the general solution for the differential equation $\frac{dy}{dx} = x^2 y - 4x^2$.

(4 marks)

b. Find the unique solution for the second order differential equation

$$y'' + 3y' - 10y = 0 \text{ where } y(0) = 5 \text{ and } y'(0) = 2$$

(6 marks)

End of Paper

APPENDIX

A. Differentiation Rules

$\frac{d}{dx}[x^n] = nx^{n-1}$; n is any real number

$\frac{d}{dx}[f(x).g(x)] = f(x)g'(x) + f'(x)g(x)$; The Product Rule

$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$; The Quotient Rule

$\frac{d}{dx}[f(g(x))] = f'(g(x)).g'(x)$; The Chain Rule

$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1}.g'(x)$; The power rule combined with the chain rule:

$\frac{d}{dx}[\sin x] = \cos x$ $\quad$ $\frac{d}{dx}[\cos x] = -\sin x$ $\quad$ $\frac{d}{dx}[\tan x] = \sec^2 x$

$\frac{d}{dx}[\sec x] = \sec x \tan x$ $\quad$ $\frac{d}{dx}[\cot x] = -\csc^2 x$ $\quad$ $\frac{d}{dx}[\csc x] = -\csc x \cot x$

$\frac{d}{dx}[e^x] = e^x$ $\quad$ $\frac{d}{dx}[\ln x] = \frac{1}{x}; \quad x > 0$

B. Basic Integration Formulas

$\int cf(x)\,dx = c\int f(x)\,dx$ $\quad$ $\int k.dx = kx + C$

$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$ $\quad$ $\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$

$\int e^x\,dx = e^x + C$ $\quad$ $\int \frac{1}{x}dx = \ln|x| + C$

Integration by-parts: $\int u\,dv = uv - \int v\,du$

Volume (disk) $= \pi\int_a^b (f(x))^2\,dx$ $\quad$ **Area** $= \int_a^b (f(x) - g(x))dx$

Volume (washer) $= \pi\int_a^b \left[(f(x))^2 - (g(x))^2\right]dx$